A Broad-Band Directional Coupler for Both Dielectric and Image Guides

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Abstract —**A four-port directional coupler is presented for use in dielectric and image guides. Simple design equations are derived and the subsequent results are in good agreement with theory. Broad-band performance has been achieved over the entire conventional waveguide bandwidth (26–40 GHz).**

I. Introduction

To date, various directional coupling structures [1]–[5] have been proposed for use in image and dielectric guide circuits. Of these, the distributed couplers tend to be narrow band because they employ evanescent field interaction which is strongly frequency dependent. An alternative is presented in this paper which is similar to the optical Fabry–Perot etalon [6] and the analysis adopts the same multiple reflection model. Rudokas and Itoh [7] have investigated a beam splitter in inverted strip dielectric guide where the desired coupling is achieved by adjusting the gap width. If a dielectric film, whose effective dielectric constant is greater than that of the guide, is used instead of a gap, a greater bandwidth is possible. This requires the correct choice of both the film dielectric constant and width but the extra design complexity is offset by the improved performance. The coupler is investigated in image and dielectric guides and should also be realizable in insular guide and inverted strip dielectric guide.

II. Theory

The reflection and transmission properties of an infinite dielectric slab, thickness D, embedded in a different dielectric medium may be derived by consideration of multiple reflections at the two interfaces [6], [8]. Let a wave, represented in Fig. 1 by a ray, be incident on the slab at an angle θ_i. A fraction r_0 of the amplitude is reflected at an angle θ_i and $1+r_0$ refracted into the slab at an angle θ_r. For a lossless dielectric, it reaches the second interface having suffered a phase delay of $\beta_2 D \sec\theta_r$, where β_2 is the propagation constant in the slab. A fraction $-r_0$ of the refracted ray is now reflected towards the first interface with $1-r_0$ transmitted out of the film and so on.

The reflection coefficient is obtained by summing all the returning rays at a front normal to their propagation direction and then referring the front to the point of incidence

$$\rho = \sum_{a=0}^{\infty} r_a$$

and similarly for the transmission coefficient

$$\tau = \sum_{b=0}^{\infty} t_b.$$

Considering the returning rays, then

$$r_1 = r_0(r_0-1)(r_0+1)\exp\{-j2D(\beta_2\sec\theta_r - \beta_1\tan\theta_r\sin\theta_i)\}$$

$$r_2 = r_0^3(r_0-1)(r_0+1)\exp\{-j4D(\beta_2\sec\theta_r - \beta_1\tan\theta_r\sin\theta_i)\}$$

$$\vdots$$

$$r_n = \frac{(r_0-1)(r_0+1)r_0^{2n}}{r_0}\exp(-j2nqD)$$

Manuscript received January 23, 1984; revised August 20, 1984.

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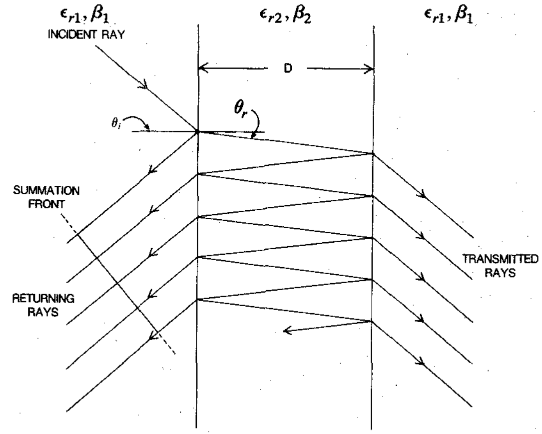

Fig. 1. Schematic of multiple reflections in a dielectric film.

where $q = \beta_2\cos\theta_r = \beta_2\sec\theta_r - \beta_1\tan\theta_r\sin\theta_i$ using Snell's law

$$\Rightarrow \rho = r_0 + \frac{(r_0-1)(r_0+1)}{r_0}\sum_{n=1}^{\infty} r_0^{2n}\exp(-j2nqD) \qquad (1)$$

and after appropriate manipulation results in

$$\rho = \frac{2jr_0\tan qD}{(1-r_0^2)+j(1+r_0^2)\tan qD}. \qquad (2)$$

Since the general case of oblique incidence is being considered, r_0 is the appropriate Fresnel reflection coefficient.

Thus

$$\rho = \frac{jA\tan qD}{B+jC\tan qD} \qquad (3)$$

$$\tau = \frac{B(1+\tan^2 qD)^{1/2}}{B+jC\tan qD} \qquad (4)$$

where

$$A = \frac{1}{(m,s)^2} - 1 \qquad (5a)$$

$$B = \frac{2}{m,s} \qquad (5b)$$

$$C = \frac{1}{(m,s)^2} + 1 \qquad (5c)$$

$$m = \left(\frac{\epsilon_{r2}}{\epsilon_{r1}}\right)^{1/2}\left(1 - \frac{\epsilon_{r1}\sin^2\theta_i}{\epsilon_{r2}}\right)^{1/2}\sec\theta_i \qquad (5d)$$

$$s = \left(\frac{\epsilon_{r2}}{\epsilon_{r1}}\right)^{1/2}\left(1 - \frac{\epsilon_{r1}\sin^2\theta_i}{\epsilon_{r2}}\right)^{-1/2}\cos\theta_i. \qquad (5e)$$

m or s are used for perpendicular and parallel polarization, respectively.

Comparison of (3) and (4) reveals that the reflected wave leads the transmitted one by 90° independently of frequency, film thickness, and dielectric constants. Fig. 2 shows the variation of $|\rho|^2$ with qD.

Maxima occur when

$$qD = (2u+1)\frac{\pi}{2}, \qquad u = 0,1,2,\cdots$$

$$\Rightarrow |\rho|^2 = A^2/C^2 \qquad (6)$$

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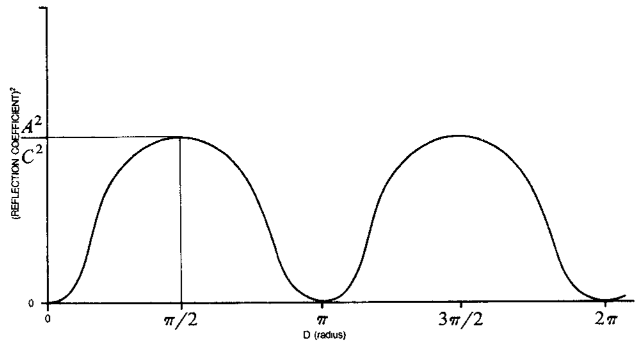

Fig. 2. |Reflection Coefficient|^2 versus qD.

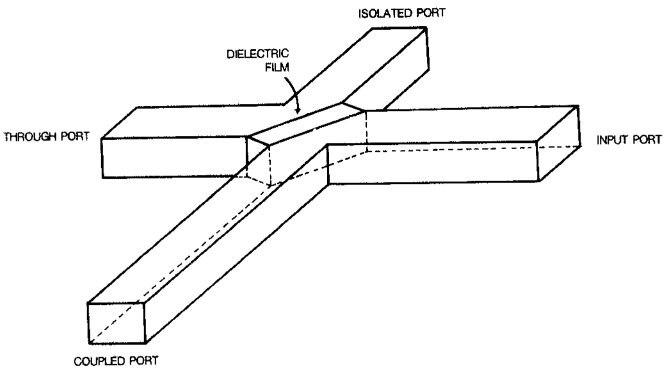

Fig. 3. Configuration of coupling structure.

and minima for

$$qD = v\pi, \qquad v = 0,1,2,\cdots$$
$$\Rightarrow |\rho|^2 = 0. \tag{7}$$

III. Design

A broad-band coupler can be constructed implementing simple design equations. Let the coupling coefficient K be defined as the ratio of reflected to incident power. Optimum performance is achieved by having the minimum possible variation of K with frequency. This occurs around a maximum in Fig. 2 and design is centered there

$$K = A^2/C^2$$
$$\Rightarrow (1-K)m^4 - 2(1+K)m^2 + (1-K) = 0$$

for perpendicular polarization.

Solving for m^2 and subsequently expressing m in terms of the two dielectric constants yield

$$\frac{\epsilon_{r2}}{\epsilon_{r1}} = \frac{(1+K \pm 2\sqrt{K})/(1-K) + \tan^2\theta_t}{\sec^2\theta_t}$$

which for $\theta_t = 45°$ has two realizable solutions

$$\frac{\epsilon_{r2}}{\epsilon_{r1}} = \frac{1}{1+\sqrt{K}} \tag{8a}$$
$$\frac{1}{1-\sqrt{K}}. \tag{8b}$$

The thickness of the film is defined for broad band by $qD = \pi/2$, i.e., $u = 0$ and using (8b)

$$\Rightarrow D = \frac{\pi}{\sqrt{2}\,\beta_2}(1+\sqrt{K})^{-1/2}. \tag{9}$$

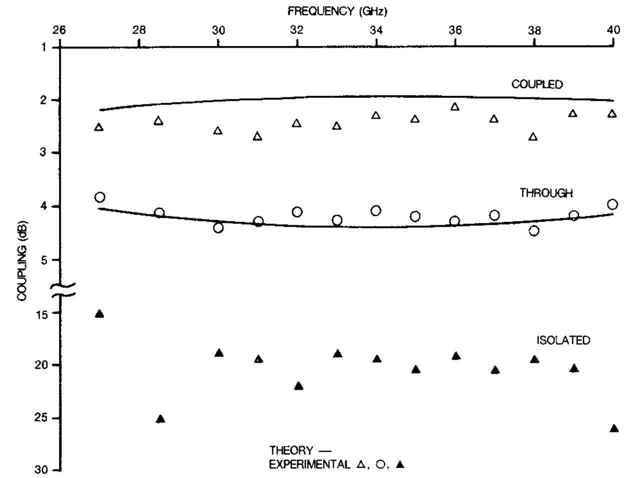

Fig. 4. Coupling coefficients versus frequency

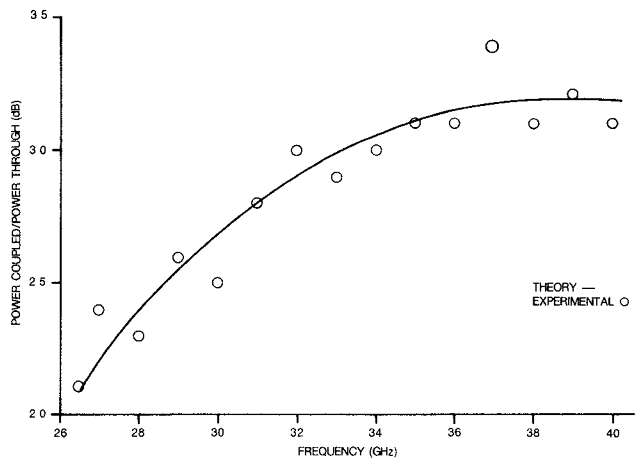

Fig. 5. Ratio of coupling coefficients versus frequency.

The forementioned considerations are applicable in image and dielectric guide (Fig. 3) provided the EM waves are well guided (Appendix A). A modification is necessary because propagation in these media is defined by an effective dielectric constant [2], [9] which is related to frequency, guide dimensions, and dielectric constant via a transcendental equation. It replaces the dielectric constant wherever encountered.

It should be noted that because of the frequency dependence of the effective dielectric constant, maxima are no longer defined by (6) since A, B, and C are not constants. However, the frequency offset between true maxima and those defined by (6) is small, the difference in magnitude being negligible. For greater accuracy, the film thickness is easily adjusted using (3).

In calculating the effective dielectric constant ratio it is best to use (8b) for the smallest refraction angle which for a finite film comes closest to (1). It also ensures that the effective dielectric constant ratio is greater than unity whatever the coupling coefficient. This does not allow a critical angle for total internal reflection at the film, and therefore transmission through it is always possible.

IV. Experimental

A 2-dB, 26–40-GHz image guide coupler was investigated using cross-linked polystyrene $\epsilon_{r1} = 2.53$ and cross section 4.8×2.4 mm as the guide. This required an alumina film with dielectric constant $\epsilon_{r2} = 9.6$ and thickness 0.625 mm. Power was launched onto the guide via a waveguide transition [10]. The insertion loss of two transitions and a straight section of guide having equal length as that used in the coupler was first measured. Thus, power at the coupler ports was corrected for noncoupler losses. The theoretical and experimental behavior of the coupler is shown in Fig. 4.

Fig. 5 illustrates the coupling ratio achieved using dielectric guide of cross section 5.5×5.5 mm and $\epsilon_{r1} = 2.1$ with the same alumina film. An isolation better than 25 dBs was achieved throughout. In both cases, the guide cross-sectional dimensions [9] were calculated to be as large as possible while still supporting only monomode propagation at 40 GHz.

V. Conclusion

The measurements show that over the entire band 26–40 GHz a 2.5-dB coupler has been constructed in image guide. The variation in coupling coefficient was within ±0.4 dB of the average value, i.e., 2.5 dB. The output at the isolated port was at least 20 dB down over this range. The theoretical model predicted a 2-dB coupling coefficient, with 4.33 dB for the through port. It was found in practice that the dielectric film in the coupler did not make perfect contact with the ground plane and this resulted in more power going to the through port and less to the coupled port. This problem did not occur in the dielectric waveguide as there is no ground plane.

The broad-band performance can be attributed to the fact that coupling takes place in the thin dielectric film, and so these couplers will be much smaller than forward-wave couplers. Also, the small size reduces the overall attenuation of the circuitry.

Appendix

The approximation that a well-guided first-order image or dielectric guide mode is equivalent to a plane wave propagating along the longitudinal axis of the guide follows directly from the Knox and Toulios [9] analysis. In this, the guiding structure is simplified to an infinite slab having an effective dielectric constant. Two plane waves can be used to represent the slab guide TE or TM modes and these will be incident on the two slab boundaries at the same angle $90-\phi°$, which is greater than the critical angle. Hence, the angles of incidence of the two waves on a dielectric film oriented at 45° to the longitudinal axis and $45+\phi°$ and $45\text{-}\phi°$, their reflection coefficients being $\rho_{45+\phi}$, $\rho_{45-\phi}$.

The overall reflection coefficient ρ_T for the mode is given by

$$\rho_T = \frac{1}{2}\rho_{45+\phi} + \frac{1}{2}\rho_{45-\phi}. \tag{A1}$$

Provided ϕ is small, which will be the case for a guide with low dielectric constant (even lower effective dielectric constant) and well-guided mode, then

$$\rho_T \approx \rho_{45}. \tag{A2}$$

Otherwise, ϕ must be evaluated from

$$\phi = \tan^{-1}\left(\frac{K_x}{K_z}\right) \tag{A3}$$

where K_z, K_x are the longitudinal and transverse propagation constants, respectively, in the slab; and Al used in the design routine.

Acknowledgment

This work has been supported by a Department of Industry grant. The authors wish to thank E. J. Griffin, R.S.R.E. Malvern, R. W. Yell, and N. P. L. Teddington who have co-supervized this project.

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An Iterative Moment Method for Analyzing the Electromagnetic Field Distribution inside Inhomogeneous Lossy Dielectric Objects

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***Abstract* — An iterative method is proposed for solving the electromagnetic deposition inside lossy inhomogeneous dielectric bodies. The technique uses the conventional method of moments to formulate the problem in matrix form. The resulting system of linear equations is solved iteratively by the method of conjugate gradients.**

The main advantage of the method is that the iterative procedure does not require the storage of any matrix, thus offering the possibility of solving larger problems compared to conventional inversion or Gaussian elimination schemes. Another important advantage is that monotonic convergence to a solution is ensured and accomplished within a fixed number of iterations, not exceeding the total number of basis functions, independently of the initial guess for the solution.

Preliminary examples involving two-dimensional cylinders of fat and muscle are illustrated. The iterative method is extendable and applicable to the three-dimensional case.

Manuscript received February 13, 1984; revised September 17, 1984. This work was supported in part by the National Science Foundation, NSF ECS 81-20305 and the Office of Naval Research, N00014-81-K-0245.

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